September 23

Random walking

In http://tieba.baidu.com/f?kz=42429501 it is asked that:

If A and B are started from same point of an axis. Each time, they throw a coin. If it is head, A will move forward by 1; otherwise, B will move forward by π. They'll not stop until the coordinate of A is larger than that of B. What is the probability that the game will stop?

In http://bbs.emath.ac.cn/thread-331-1-1.html, the problem is analyzed and finally I got the result that the probability is between 0.54364331210052407755147385529445 and 0.54364331210052407755147385529454.

If B moving forward by another number x instead of π, and the probability that the game will stop is p(x), it is easy to know that p(x) is monotone decreasing function. If we find two rational numbers a and b which satisfy $a>\pi>b$, we have $p(a)<p(\pi)<p(b)$. This means as soon as we could solve the problem for all rational number x, we could find the approximate value of $p(\pi)$.

For any rational number x, we could transform the problem into a version that A moving forward by integer m and B moving forward by integer n (where n is larger than m). In 5#,this problem is analyzed. Let's assume that the probability is $q(k),(k\geq1)$ given A is after B by k. We have linear recurrence equation $q(k)=\frac{1}{2}q(k-m)+\frac{1}{2}q(k+n),(k\geq1)$ and the correspondent characteristic polynomial is $x^{m+n}-2x^m+1=0$. In http://bbs.emath.ac.cn/thread-332-1-3.html, Rouché's Theorem is used to show that there're exact m roots of the polynomial whose norms are less than 1(And n-1 roots whose norm is larger than 1). Let's assume the m roots whose norms are less than 1 are $x_1,x_2,\dots,x_m$ while other roots are $x_{m+1},\dots,x_{m+n}$. q(n) could be written as $q(k)=a_1x_1^k+a_2x_2^k+\dots+a_{m+n}x_{m+n}^k$. Since $0\leq q(k)\leq1$, and it is easy to prove that q(k) goes to 0 as k goes to infinity. According to analysis in http://bbs.emath.ac.cn/thread-354-1-1.html, the cofficients $a_{m+1},a_{m+2},\dots,a_{m+n}$ should be 0 and so that we have $q(k)=a_1x_1^k+a_2x_2^k+\dots+a_mx_m^k$.

Further analysis on 18# shows $q(n)=1+\prod_{k=1}^{m}(1-y_k)$ where $y_i=\frac{1}{x_i}$ and p(x) is $\frac{1}{2}+\frac{q(n)}{2}$.

11# shows p(π) is between 0.54364331210052407755147385529445 and 0.54364331210052407755147385529454.

In 23# zgg draws the picture of p(x) with the algorithm above.

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